Archery

This problem gives you the chance to:
• draw a box plot
• compare sets of data

Guy and Sagar both enjoy archery and hope to be picked
for their college team.

There have been 15 matches in college this year.
These are the scores for Guy.

| 1192 | 1258 | 1038 | 1208 | 956 | 1052 | 1262 | 994 | 1128 | 1066 | 1286 | 1174 | 1050 | 926 | 1240 |

Guy's mean score is 1122.

These are the scores for Sagar.

| 1134 | 1098 | 1182 | 1126 | 1066 | 1204 | 1052 | 1072 | 1156 | 1102 | 1088 | 1220 | 1168 | 1106 | 1164 |

Sagar's mean score is 1129.

Here is a box plot for Guy's scores.
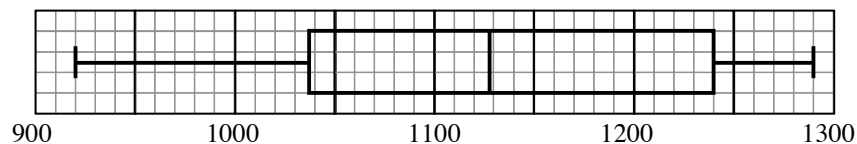
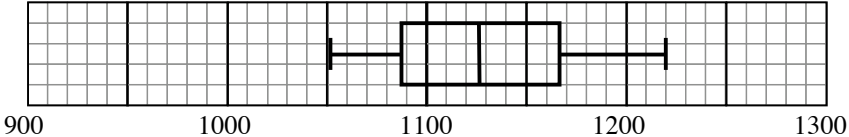1. Draw a box plot for Sagar's scores.
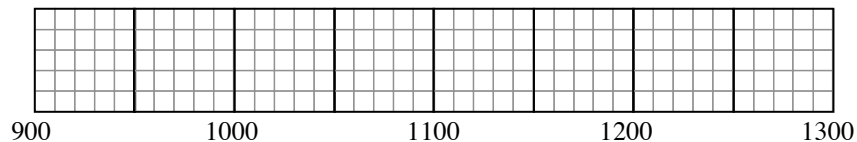
2. Explain the main points on your box plot.

_____

_____

_____

3. Who is the more consistent archer?
   Explain how you know.

                                                            _____

_____

_____

_____

_____

4. If you were picking the college team would you choose Guy or Sagar?
   Explain why you would make this choice.

                                                            _____

_____

_____

_____

_____

9

Archery		Rubric
The core elements of performance required by this task are: • draw a box plot • compare sets of data Based on these, credit for specific aspects of performance should be assigned as follows	points	section points

		points	section points
1.	Draws a correct box plot: Minimum and maximum correct. (1052, 1220)	1	
	Lower quartile correct: (1088 or 1093) and upper quartile: (1168 or 1166)	1	
	Median correct (1126)	1	
			3
2.	Explains that: the maximum and minimum points are Sagar's highest and lowest scores.	1	
	the box corresponds to the quartiles	1	
	with the median indicated	1	
			3
3.	Gives correct answer: **Sagar**	1	
	Gives a correct explanation such as:		
	The range and interquartile range of Sagar's scores are much smaller than those of Guy.	1	
			2

4.	Gives correct answer: **Sagar**
	                    and explains that Sagar is more consistent.
	                    Or has a higher mean.
		1	

	or	or	

	Gives correct answer: **Guy**
	                    and explains that Guy sometimes gets very high
	                    scores which might win them the match.
		1	

			1
	Total Points		**9**

Archery

Work the task and look at the rubric. What are the big ideas about data and measures of center that a student needs to understand to be successful on this task? _____

Look at student work on part 1, making a box plot and refer to part 2 for supporting evidence.
- Do you see evidence that students ordered the data before making the graph?
- Do you see evidence that students found and could plot the correct low and high scores?_____
- Were errors on high and low scores attributable to:
    - Problems with reading scale?_____
    - Not understanding the connection between high and low to the structure of the box plot?_____
    - Choosing incorrect values?_____
- How many students could identify and plot the median?_____
- How many students used the mean as the midline on the graph?_____

How much experience do your students have with making and reading box and whisker graphs? Why are these useful for making sense of the data?

Now look at student work on part 3, which archer is most consistent. How many of your students thought that Sagar was most consistent because:
- He had the lowest range?_____
- He had the highest or best scores?_____
- He had a higher mean?_____
How many of your students picked Guy because:
- His scores are closer? _____
- His box plot is larger? _____
- Other? _____

Now look at work on part 4, who should the team pick. How many of your students:
- Picked Sagar because he is most consistent?_____
- Picked Sagar because he will get an okay score?_____
- Picked Sagar because he has a higher mean?_____
- Picked Guy because he had higher scores?_____
- Picked Guy because he is more consistent?_____
- Other?

What are the most important ideas about data that your students need clarified? What might be your next steps?

Looking at Student Work on Archery

Student A orders the data and is able to make an accurate box plot. The student is able to name the important parts of the box and whisker plot and explains how to find the median and interquartile points. The student is able to identify that Sagar is most consistent and explain why that is important in part 5.
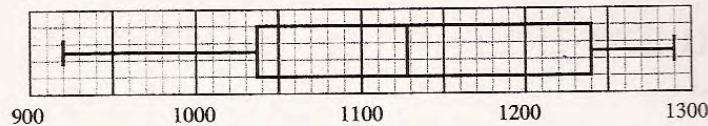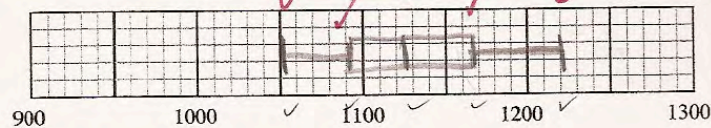
Student A

These are the scores for Sagar.

| 1134 | 1098 | 1182 | 1126 | 1066 | 1204 | 1052 | 1072 | 1156 | 1102 | 1088 | 1220 | 1168 | 1106 | 1164 |

1052   1066   1072   1088   1098   1102   1106   1126   1134   1156
1164   1168   1182   1204   1220

Sagar's mean score is 1129.

1166

1088   1098

1093

Here is a box plot for Guy's scores.

1. Draw a box plot for Sagar's scores.

Student A, part 2

2. Explain the main points on your box plot.

the First Point was the lowest score and the
third,
~~Second~~ point was the median between the First and last
point. the last point was the largest score. the second and
third points are the median of the two halfs

3. Who is the more consistent archer?
   Explain how you know.

Sagar

Sagar was the more consisten archer because his
box plot was not as wide as Guy's so his
scores were all around the same

4. If you were picking the college team would you choose Guy or Sagar?
   Explain why you would make this choice.

Sagar

I would choose Sagar because he was more
consistant than Guy. Guy might make a relly bad
shot but Sagar could be counted on to
make an ok shot.

Student B is able to order the data and find the median. The student is unable to use this information to accurately plot the points on the graph. The student does not understand the scale and the "between spaces" on the graph. Notice that all the points fall on the grid lines. Student B gives a legitimate reason for picking Guy over Sagar.
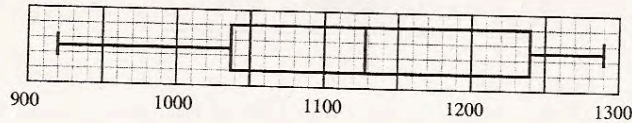
Student B

These are the scores for Sagar.

| 1134 | 1098 | 1182 | 1126 | 1066 | 1204 | 1052 | 1072 | 1156 | 1102 | 1088 | 1220 | 1168 | 1106 | 1164 |

1052, 066, 1072, 1088, 1098, 1102, 1106 (1126) 1134, 1156, 1164, 1168 1182, 1204, 1220

Sagar's mean score is 1129.

Here is a box plot for Guy's scores.

1. Draw a box plot for Sagar's scores.
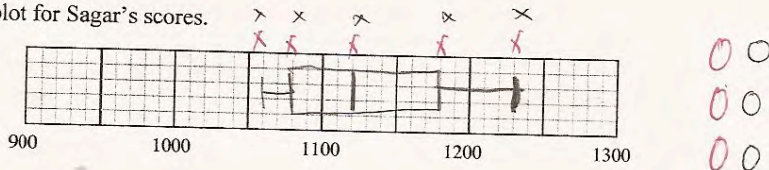
2. Explain the main points on your box plot.

The main plots on my box are the high and low number the median and the first and third quartile

3. Who is the more consistent archer?
   Explain how you know.

Sagar

He has not so much a lot of range from the lowest score to the highest. So that is more consistent to me.

4. If you were picking the college team would you choose Guy or Sagar?
   Explain why you would make this choice.

Guy

I would pick Guy because he has one of the most highest score between him and sagar.

Student C understands that the data needs to be ordered, but is unsuccessful in accomplishing that task. The student knows that the median is the center of the graph but like Student B does not understand scale and the "between spaces". The student should not have been given credit for the rationale in part 3. High scores do not make Sagar consistent. And being consistent is not a reason for being consistent. *How do we help students develop the logic of making a convincing argument? What did the student need to understand about the term consistent and the about data to be successful on this part of the task?* The student does notice that Guy has several scores lower than Sagar's.

Student C

These are the scores for Sagar.

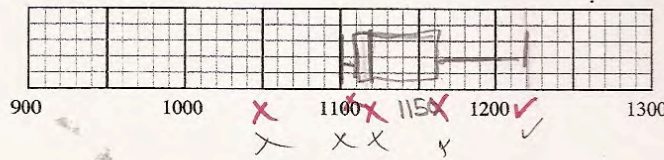| 1134 | 1098 | 1182 | 1126 | 1066 | 1204 | 1052 | 1072 | 1156 | 1102 | 1088 | 1220 | 1168 | 1106 | 1164 |
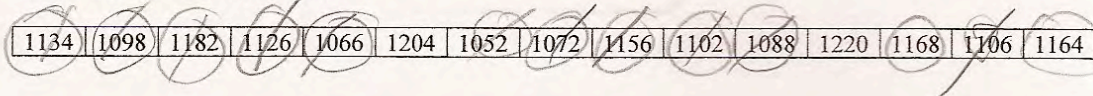
Sagar's mean score is 1129.

1106, 1052, 1066, 1072 / 1088, 1098, 1102, 1126, 1134, 1156, 1164, 1166,
1182, 1204, 1220
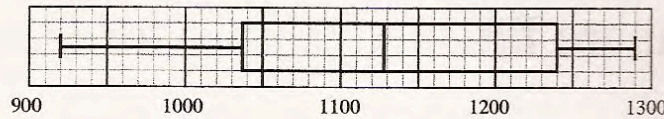
Here is a box plot for Guy's scores.

1. Draw a box plot for Sagar's scores.

Student C, part 2

2. Explain the main points on your box plot.

Saga's median was 1126. His lowest score was 1106 and his highest score was 1220.

3. Who is the more consistent archer?
   Explain how you know.

Sagar

He has the highest scores and they are very consistant.

4. If you were picking the college team would you choose Guy or Sagar?
   Explain why you would make this choice.

Sagar

Because it would be more consistant and help the team becaus he has never gone under 1100.

Student D does not show evidence of ordering the data. The description in part 2 shows that the student has the idea of high, low and median but has a misconception of the interquartile values. Look at the response to part 3. *What experiences does this student need? What question might you pose to get the student to focus more on the meaning of the data?*

Student D

1. Draw a box plot for Sagar's scores.

900   1000   1100   1200   1300

2. Explain the main points on your box plot.

The first one is the least amount then the second is for the second least amount. Then the third is the mean and the second to the highest and the fifth is the highes

3. Who is the more consistent archer?
Explain how you know.

Both

Because they both have the same amount of scores.

4. If you were picking the college team would you choose Guy or Sagar?
Explain why you would make this choice.

Sagar

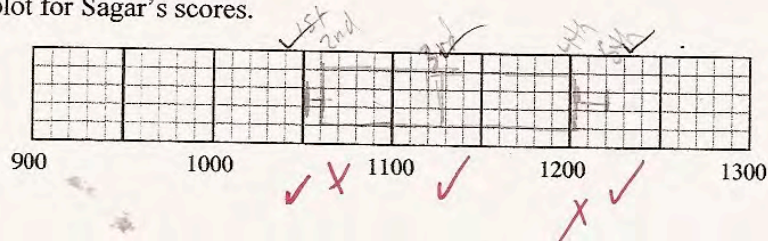Sagar because she has probably practiced more and she has better scores.

Student E has lots of partially correct ideas about data and box and whisker plots, but not enough of any one thing to be successful. The student orders the data, but doesn't use it to make the box and whisker plot. The student knows the names of the five features that make up the box plot. The student uses the size of the graph rather than the data for minimum, maximum and median (although the median is not the point graphed). The student has probably given the correct values for upper and lower quartile in the table, but graphs something different. The student doesn't understand the term consistency in a mathematical context.
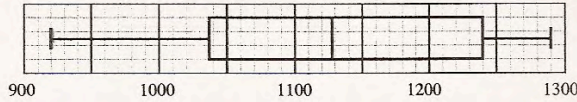
Student E

These are the scores for Sagar.

| 1134 | 1098 | 1182 | 1126 | 1066 | 1204 | 1052 | 1072 | 1156 | 1102 | 1088 | 1220 | 1168 | 1106 | 1164 |

Sagar's mean score is 1129.

1052, 1066, 1072, 1088, 1098
1102, 1106, 1126, 1134, 1156, 1164, 1168, 1182
1204, 1220

Here is a box plot for Guy's scores.
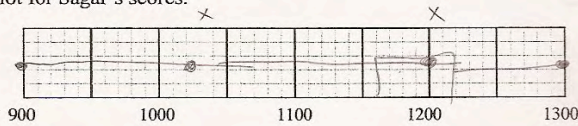
900   1000   1100   1200   1300

1. Draw a box plot for Sagar's scores.

900   1000   1100   1200   1300

minimum   lower   median   upper   maximum
900   1088   1100   1106   1300

Page 6   Archery

2. Explain the main points on your box plot.

1. minimum 2 maximum 3. lower 4. upper
5. median

3. Who is the more consistent archer?
   Explain how you know.

   sagar

   I picked sagar because he
   had the higher mean score

4. If you were picking the college team would you choose Guy or Sagar?
   Explain why you would make this choice.

   sagar

   he had more higher mean

Student Task	Draw a box plot and compare sets of data.
Core Idea 5 Statistics	**Understand statistical methods used to display, analyze, and compare and interpret different data sets.** • Construct the most appropriate graph, including frequency distribution, stem and leaf, and scatterplot, for a given set of data and justify the choice. • Analyze data, including finding measures of center and spread presented in a frequency distribution. • Find, use and interpret measures of center and spread, including interquartile range.

Mathematics of this task:
- Understanding definitions and using of measures of center: mean, median, and range and applying them in context
- Making a box and whisker plot from data
- Examining different measures of center and determining which is more important for making a decision

Based on teacher observations, this is what seventh graders knew and were able to do:
- Identify the minimum and maximum numbers
- Recognize which archer is most consistent and give a supporting reason
- Choosing someone to be on the team and give a supporting reason

Areas of difficulty for seventh graders:
- Recognizing a need to order the data
- Confusing the midline on the box as mean instead of median
- Understanding scale and estimating the between spaces
- Graphing a box and whisker plot
- Using and interpreting academic language for data

Task 4 - Archery

Mean: 3.03   StdDev: 2.59

Table 38: Frequency Distribution of MARS Test Task 4, Grade 7

Task 4 Scores	Student Count	% at or below	% at or above
0	777	16.3%	100.0%
1	959	36.3%	83.7%
2	675	50.4%	63.7%
3	696	65.0%	49.6%
4	458	74.6%	35.0%
5	315	81.2%	25.4%
6	257	86.5%	18.8%
7	213	91.0%	13.5%
8	216	95.5%	9.0%
9	214	100.0%	4.5%

Figure 47: Bar Graph of MARS Test Task 4 Raw Scores, Grade 7

MARS Task 4 Raw Scores
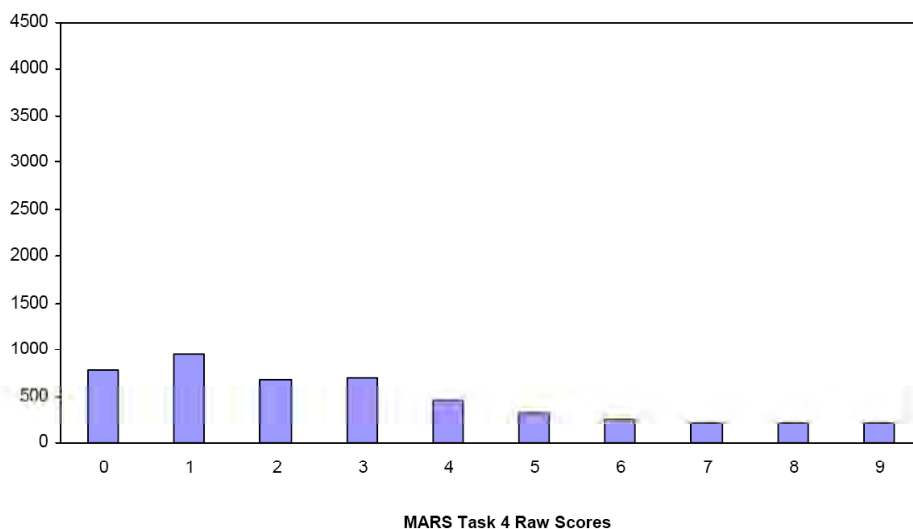
The maximum score available for this task is 9 points.
The minimum score needed for a level 3 response, meeting standards, is 4 points.

Most students, about 83%, were able to identify Sagar as the most consistent archer. Almost half the students could identify Sagar as the most consistent, use range to explain the choice, and pick a player for the team using data to support the choice. Some students, about 35%, could also identify one of the important features of the box and whisker plot (high and low score or median). A few students, less than 5%, could meet all the demands of the task including making an accurate graph and identifying all the features of a box and whisker plot. 16% of the students scored no points on the task. 82% of the students with this score attempted the task.

Archery

Points	Understandings	Misunderstandings
0	82% of the students with this score attempted the task.	Students thought Guy was the most consistent because he had a larger box plot or higher scores.
1	Students picked Sagar as most consistent, but didn't give correct reasoning for that choice.	11% thought Sagar was most consistent because he had a higher mean. Some students thought Sagar had higher scores. Students had difficulty comparing the data sets and giving a reason for someone to be on the team.
3	Students knew that Sagar was more consistent because the range was smaller and could compare the data to choose someone for the team.	Students had difficulty identifying the major points on the box plot. 10% did not attempt part 2 of the task. 21% did not mention mean and interquartiles. 26% did not mention minimum and maximum. 26% used mean for the middle value.
4	Students could reason about consistency and choose a player for their team. Students could name one of the important features of a box plot.	Students did not know to order data before making the plot. If they did order the data, they had trouble interpreting scale and often rounded values to even grid lines. Students were unclear how to find the major locations for making the box and whisker plot.
9	Students could reason about consistency and choose a player for their team. Students could name, quantify, and locate important features of a box plot on the graph.	

Implications for Instruction

Students need to learn about data in context, so they develop an appreciation of how the measures are used. Using data in context allows students to have a purpose for using academic language and helps them develop richer meanings for the terms. They need to understand the purpose of the various measures of center. This helps to clarify procedures. For example, if the student understands the range is looking at how far apart the data points are and has opportunities to see where data is close together or spread apart, then they will be less likely to use the largest value for range or add together the largest and smallest data points.

Students are hurt by learning short, quick definitions of terms. Many students have a shortened definition for median as the number in the middle. This ignores the importance of first ordering the numbers.

Data should be used to help answer questions or make decisions. Working in a context helps students to see a purpose for looking at the measures of centers to make sense of the question or decision being solved. By working in context, hopefully students will develop a sense that the measures are trying to describe or typify the numbers in the data set. Looking at and comparing similar data helps them realize how the measures describe the similarities and differences in the data sets. These nuances of meaning don't come into play when just working with strings of numbers.

Students need to see how the box and whisker plot gives a picture about the shape of the data. Students should be able to notice how the shapes vary between data sets and discuss how that variation effects decision making.

Students at this grade level still need to work on scale. As students move up in the grades, the span between grid lines becomes larger and more difficult to divide up. It is important for students to be exposed to a wide variety of scales and discuss what the un-labeled grid lines represent and what might be the value of the numbers between the grid lines.

Ideas for Action Research

Re-engagement – Confronting misconceptions, providing feedback on thinking, going deeper into the mathematics. *(See overview at beginning of toolkit).*

1. Start with a simple problem to bring all the students along. This allows students to clarify and articulate the mathematical ideas.
2. Make sense of another person's strategy. Try on a strategy. Compare strategies.
3. Have students analyze misconceptions and discuss why they don't make sense. In the process students can let go of misconceptions and clarify their thinking about the big ideas.
4. Find out how a strategy could be modified to get the right answer. Find the seeds of mathematical thinking in student work.

One of the key ideas of this task is that students had difficulty using scale. They might identify the correct numbers, but be unable to locate the values on the graph. Students seemed to want to use only the grid lines for locating points. To start a re-engagement lesson, the teacher might give students some blank grids and ask them to locate points. The teacher might give a value and then show a graph with the incorrect location. For example: *"I overheard someone say that the lowest value was 1052 and the student made this graph:*

These are the scores for Sagar.

| 1134 | 1098 | 1182 | 1126 | 1066 | 1204 | 1052 | 1072 | 1156 | 1102 | 1088 | 1220 | 1168 | 1106 | 1164 |

Sagar's mean score is 1129.

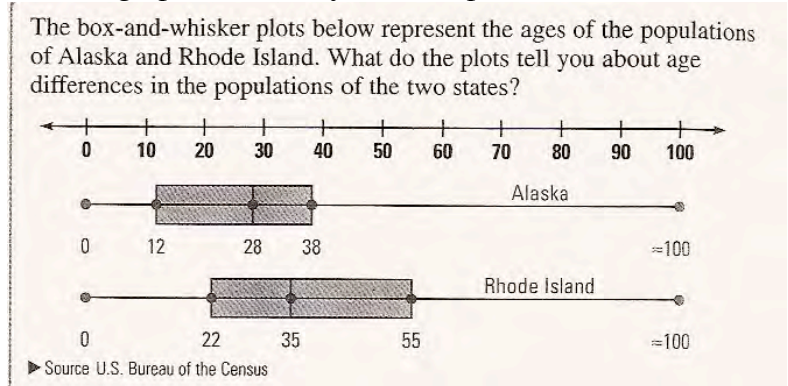Here is a box plot for Guy's scores.

1. Draw a box plot for Sagar's scores.

Why do you think the scorer marked this wrong? Where should the mark go? During the discussion students will need to discuss what part of the graph corresponds to the low number as well as talk about the issue of scale.

The teacher might next explore why the student has rearranged the numbers below the data box. *What is the student doing? Why? Why is the number marked wrong on the graph? Where should the number go?*

After exploring the information about scale, the teacher might choose to have students discuss the importance of the graph and what information it gives. One way to really see how the graph tells a story is to use plots like the ones below:

The box-and-whisker plots below represent the ages of the populations of Alaska and Rhode Island. What do the plots tell you about age differences in the populations of the two states?

Alaska

Rhode Island
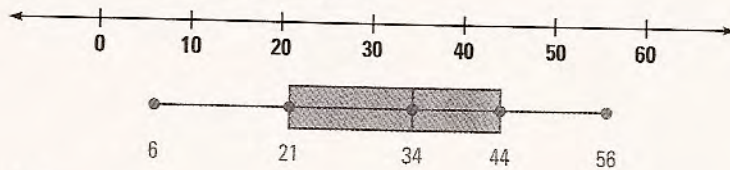
▶ Source U.S. Bureau of the Census

The box-and-whisker plots represent speeds (in miles per hour) of cars at midnight and noon on one city street. Which plot is more likely to represent noon?
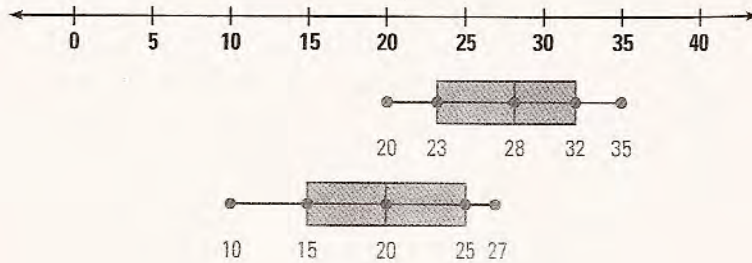
By comparing different data sets for similar information, noticing the distributions gives an immediate idea about differences and their significance.

The teacher might present the class with a box and whisker plot and tell the students the number of data points and ask them to come up with possible data points. For example: Below is box and whisker plot of some video game scores. *If there were 15 players, what might be their scores? How do you know? Do you think everyone will have the same numbers? How might they be different? How do you think they will be the same?*

Graphs taken from Mathematics Concepts and Skills, Course 2 *published by McDougal Littell.*

Performance Assessment Task
Archery Grade 7 task aligns in part to CCSSM HS Statistics & Probability
The task challenges a student to demonstrate understanding of statistical methods used to display, analyze, compare, and interpret different data sets. A student must understand how to construct the most appropriate graph, including frequency distribution, stem and leaf, and scatter plot, for a given set of data and justify the choice. A student must analyze data, including finding measures of center and spread presented in a frequency distribution. A student must make sense of the data representation by being able to find, use and interpret measures of center and spread, including interquartile range. In this case, a student must make sense of the constraints of the problem situation in order to make a mathematical argument and justification for the selection of a particular archer to join the college archery team.
Common Core State Standards Math - Content Standards
High School – Statistics and Probability – Interpreting Categorical and Quantitative Data **Summarize, represent, and interpret data on a single count or measurement variable.** S-ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).
Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.5 Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the

results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
7	2009	9	4	38 %